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# A GENERIC MATHEMATICAL MODEL OF GROUNDROLL MOTION OF A HIGH SPEED TRANSPORT AIRPLANE

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## ABSTRACT

A mathematical model of the 'pilot - airplane - undercarriage - operational conditions' system developed for a high speed transport airplane is introduced. The algorithms of the airplane flight dynamics, undercarriage-runway interaction, and human pilot situational decision making are briefly described. The overall objective is to develop an affordable engineering tool for examination of complex system behaviors in the groundroll and adjacent phases of flight on a computer during design.

## INTRODUCTION

When the undercarriage system is in contact with the runway, the aircraft dynamics significantly differ from the airborne flight dynamics. During groundroll the forces of interaction between the undercarriage system and the runway significantly change the airplane's dynamic properties and the control tactics required. For this reason, even a minor piloting error may result in dangerous excursions on the runway often leading to an incident or accident. Such cases are typical to those aircraft which have a large offset of the nose undercarriage unit to the CG. High speed transport airplanes belong to this group.

After a touchdown on landing, such a vehicle may exhibit directional instability, and the pilot must keep the vehicle within the runway side limits by applying both rudder and nose wheel steering inputs. However, large deflection angles of the steerable wheels may cause the wheel system to skid which results in a loss of the airplane directional stability.

## PROBLEM

The main difficulty in controlling the airplane during groundroll is that the pilot must address meet three mutually excluding requirements.

1. In order to counter large disturbances of the aircraft attitude appropriately large control inputs are needed.
2. To maintain the airplane directional controllability, much smaller inputs are required.
3. Another problem is that the airplane's dynamic properties on the runway may vary significantly due to airspeed and runway surface conditions. These and other factors change the boundaries of wheel skidding regimes. For this reason during groundroll it is difficult for the pilot to keep control inputs within varying safety limits.

As a result, the probability of pilot error at takeoff and landing increases, thus increasing the chances of an accident. A large number of accidents and incidents with transport aircraft on the ground is attributed to the aforementioned circumstances [1-5]. Thus, the research problem can be briefly formulated as follows: how to examine the system behavior and control tactics for a high speed transport airplane under complex groundroll conditions?

## SOLUTION APPROACH

Many of the groundroll modes cannot be reproduced in flight tests. The reasons are safety constraints and impossibility to achieve a required combination and level of complex operational conditions for testing.

Autonomous mathematical modeling and computer simulation techniques can be used to address this problem [9]. The main physical phenomena, which constitute the behavior of the "pilot – airplane – undercarriage – operational conditions" system, are described in a comprehensive mathematical model. This allows to reproduce and study the effects of various key design and operational factors, including onboard system failures and adverse weather conditions, on a computer.

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As a result, airplane flight testing and certification can be performed virtually on a PC at much lower cost and within a shorter time [9]. The overall objective is to obtain more knowledge of the airplane flight dynamics and piloting tactics during the runway modes in advance, before a test article is built.

## MODELED PHENOMENA

To obtain an adequate mathematical model of the airplane ground maneuvering the following physical phenomena and relationships should be accounted for:

- changes of the normal, longitudinal and side forces of interaction between the wheels and the runway surface [6]
- changes of the airplane position with respect to the runway, taking into account the wheels' elasticity
- wheels aquaplanning or skidding on the runway covered with precipitation (water, sleet, snow)
- action of the side forces generated by freely-castoring wheels due to wheels inclination
- effects of the internal wheel (tire) pressure, wheel pairs and bogie arrangement on the wheel-runway interaction forces
- effects of the wheel brake control laws and a wheel antiskid system on the reaction forces
- regimes of reverse reaction of the steerable wheels in the nose undercarriage unit, given a kinematic link between rudder and the steerable wheels
- kinematic interaction between the airplane and two external media (the airflow and the runway) as a result of their relative motion
- pilot control inputs during the transition from groundroll to airborne modes of flight and back
- effects of various conditions of the runway surface on the undercarriage reaction forces
- effects of dynamic air cushion created between the runway and the airplane during groundroll
- effects of thrust reversing on the vehicle aerodynamics and flight dynamics
- action of the moments of resistance to lateral and rolling motion modes and the stabilizing and damping moments which are created in the undercarriage system
- effects of failures in the steerable wheel control, damping and brake systems on the undercarriage reaction forces
- wheel bounces off the runway at touchdown on landing and, possibly, during takeoff
- effects of the fuselage and undercarriage system elasticity on the reaction forces.

The subject of modeling and simulation is the behavior of the "pilot – airplane – undercarriage – operational conditions" system in complex (multi-factor) flight situations. The latter include demanding flight test modes, flight accidents and incidents, including a "chain reaction" type cases, and other non-trivial situations which may require a thorough quantitative analysis. The mathematical models of flight, which have a capacity to address this kind of problems without a human pilot in the modeling loop, are called autonomous situational models [8, 9].

The assumptions of the model include the following:

- only static deformations of the airframe are accounted for
- the runway is a non-deformed, flat surface
- in the undercarriage model a hypothesis of stationary character of deviation angles of elastic wheels is employed
- a stationary aerodynamics hypothesis is employed
- the wind distributions along and across the airplane are assumed linear.

## MAIN ALGORITHMS

Thus, a model of the "pilot – airplane – undercarriage – operational conditions" system behavior includes the following main algorithms (**Fig. 1**):

- equations of motion of the airplane as a rigid body system (a flight dynamics model)
- a model of the human pilot decision making, based on a specified control tactics and combined with a model of a flight situation under examination [8]
- a model of the airplane's undercarriage system
- models of various runway surface conditions
- a model of the airplane aerodynamics (forces and moments), including the effects of engine's forward and reverse jet streams [7]
- a model of the airplane flight control system
- models of the automatic takeoff and landing control system (if present)
- a model of the effects of engine thrust on the aircraft flight dynamics
- models of the onboard system failures which affect the aircraft flight dynamics
- wind and turbulence models
- airframe elasticity models
- models of rain and icing conditions
- auxiliary algorithms.

The algorithms, which implement the models of the undercarriage-runway interaction and the human pilot decision making, are introduced below. A generic computational algorithm of a flight dynamics model is presented in Appendix.

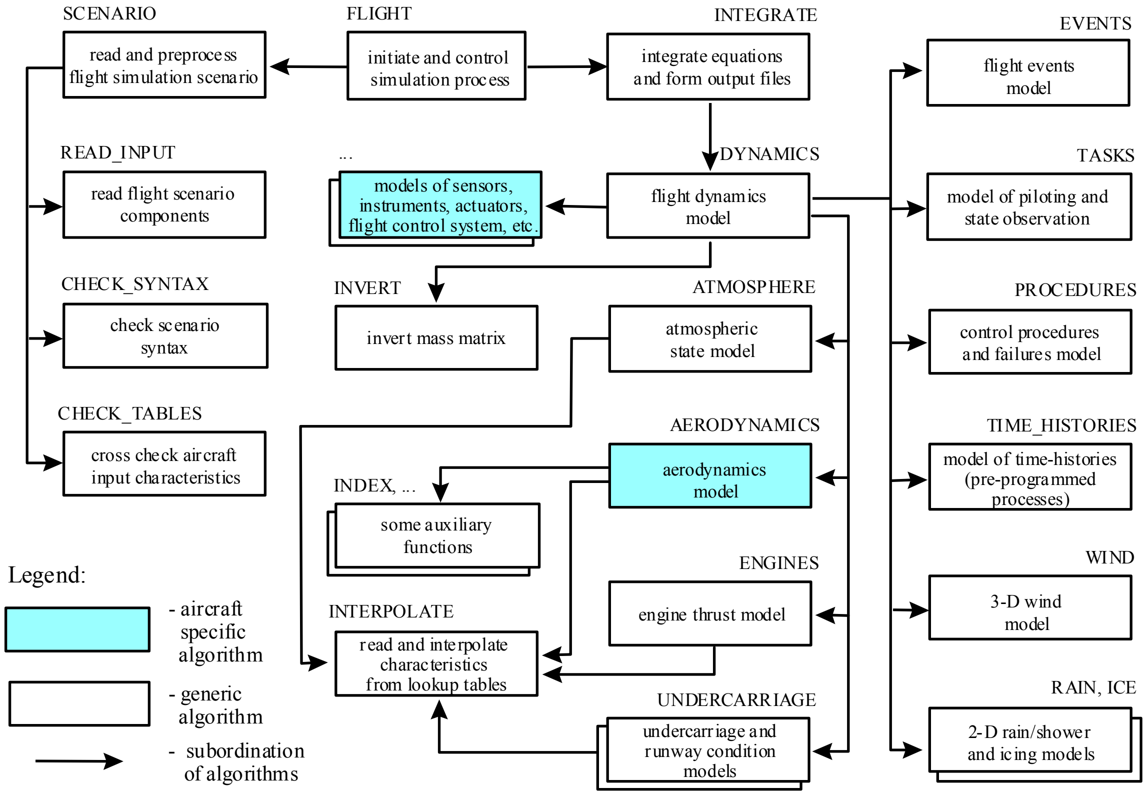


**Fig. 1**. Structure of the autonomous flight situation model

## INTRODUCTION TO THE UNDERCARRIAGE MODEL

This algorithm implements a physical mechanism of creation and action of the undercarriage reaction forces and moments as a function of the airplane attitude (with respect to the runway) and other design and operational variables. Due to a large number of equations constituting the algorithm, only a brief introduction is made below. All the computational steps are executed for each unit of the undercarriage system.

1. Given the length of the undercarriage unit (with respect the C.G.) and the rate of its change, calculate the forces of static resistance of the compressed nitrogen in the unit's shock absorber and the forces of hydrodynamic resistance to the rod motion with respect to the absorber cylinder;

2. Calculate the normal load per one wheel and the forces of wheel rolling resistance for the bogie in total;

3. For each wheel in the bogie (only for main units, i.e. for ones which are equipped with non-steerable wheels), calculate the projection of the radius-vector which links the airplane's C.G. and the center of a wheel's contact spot on the runway;

4. Calculate the DCM between the wheel axes and the runway axes (only for undercarriage unit(s) with steerable wheels);

5. For each wheel in the bogie (only for the units with steerable wheels), calculate the projection of the radius-vector, which links the airplane's C.G. and the center of a wheel's contact spot on the runway;

6. For each wheel in the bogie, calculate projections of the velocity of the center of a wheel contact spot on the aircraft body axes at zero rate of tire deformation;

7. For each wheel in the bogie, calculate the projections of the velocity of the center of a wheel contact spot on the wheel reference axes;

8. For each wheel in the bogie, calculate the speed of wheel aquaplanning as a function of internal tire pressure;

9. For each wheel in the bogie, given a required thickness of precipitation on the runway, calculate the increment of a relative coefficient of the wheel rolling

resistance force due to forward advancement of the wheel contact spot; calculate the wheel normal reaction;

10. For each wheel in the bogie, calculate the wheel friction force created when the airplane speed exceeds the aquaplanning speed;

11. Given: (a) the time instants of application and release of the wheel braking torque, (b) time delays in the wheel brake system during the brakes application and release, (c) longitudinal and lateral coefficients of the wheel-runway adhesion force as a function of the aircraft speed (the coefficients are measured using a special ground equipment), calculate, for the entire wheel system in the bogie, the degree of application of the maximum braking torque, the rate of its change, and the wheel-runway adhesion (coupling0 factor for a given aircraft ground speed;

12. Given the above conditions, calculate, for the entire wheel system in the bogie, the wheel friction forces of the braked wheels for the following cases of operation of the anti-skid system: the system is 'on', the system is 'off', and the system is in an 'on-off'/'off-on' transition mode; calculate the average friction force created by the wheel antiskid system;

13. For the wheel system in the bogie, calculate the derivative of the lateral force on the wheels due to wheel 'heading' angle at a zero wheel 'heading' angle as a function of the wheel diameter, internal tire pressure, vertical displacement and the friction force at the center of the wheel contact spot;

14. For each wheel in the bogie, calculate the wheel longitudinal stiffness as a function of the wheel diameter, internal pressure and vertical deformation;

15. For each brake wheel calculate the rate of change of the braking torque and braking force, given the calculated degree of application of the maximum braking torque (see Step 11);

16. For each brake wheel determine the rate of change of the wheel longitudinal deformation during braking;

17. If the aircraft speed is below the speed at which non-holonomic oscillations of elastic wheel have essential effects on the wheel lateral force, calculate the right part of a differential equation, which gives the rate of change of the wheel lateral deformation, based on the rolling elastic wheel hypothesis;

18. For higher groundroll speeds calculate the wheel longitudinal deformations based on a hypothesis of a stationary 'heading' angle of the elastic wheel;

19. Calculate the wheel 'heading' angles for all the wheels; in addition, if the steerable wheels are in a self-castoring mode (with damping), account for the effect of wheel inclination upon the wheel 'heading' angle;

20. Calculate the stabilizing moments on the wheels and the bogie and the moments of resistance to lateral and rolling motion of the airplane on the runway;

21. Calculate the radius-vectors and their components for all the points of application of the undercarriage reaction forces in aircraft body axes;

22. Calculate the components of the undercarriage reaction forces and the moments of these forces in body axes for the DYNAMICS algorithm (see Appendix):
$\{ F_{x\,lg}, F_{y\,lg}, F_{z\,lg}, M_{x\,lg}, M_{y\,lg}, M_{z\,lg} \}$.

## INTRODUCTION TO THE SITUATIONAL PILOT MODEL

DEFINITION. The situational human pilot model is a system of computational algorithms and input data structures that imitates a limited subset of a human pilot's knowledge and decision making functions required for situational (tactical) flight control [8, 12].

ASSUMPTIONS. The assumptions and limitations of the model are as follows:

- piloting is described as a multi-stage decision making process based on a given scenario
- control scenarios formalize the pilot's tactics at the level of cause-and-effect relationships between flight events and flight processes
- pilot's sensor and motor functions are not modeled
- pilot's strategic decision making functions are not modeled with the exception of flight scenario planning
- piloting inputs are applied by increments with a delay and error depending on the pilot's parameters
- the magnitude of these increments depend on the error between the current and desired values of the observed state variables
- the frequency of control and observation depends on the rate of change of observed state variables and required accuracy of control
- zones of the pilot's insensitivity to observation errors are introduced for each observed variable.

MAIN CONCEPTS. Three main concepts are used in the model, these are the flight event, the flight process, and the flight scenario. These concepts provide a universal language for formalization of the content and logic of various phases, modes and conditions of flight in autonomous simulation.

FLIGHT EVENT. The flight event (**E**) is a special state of the "pilot - vehicle - operational conditions" system, which is important to the pilot and stands for a substantial change in flight. Examples of flight events are as follows: $\mathbf{E}_8$: *"left engine out"*, $\mathbf{E}_2$: *"speed VR achieved"*, $\mathbf{E}_{11}$: *"altitude 360 ft and speed 180 kn"*, $\mathbf{E}_9$: *"on the runway"*, $\mathbf{E}_1$: *"high angle of attack"*, $\mathbf{E}_{14}$: *"30$^o$ left bank"*, $\mathbf{E}_6$: *"go-around decision*".

The list of all the events which may occur in a particular flight situation or in a group of situations, is called the flight event calendar, $\Omega(\mathbf{E})$. The flight event calendar may be viewed as a discrete logical framework of flight. Flight events are depicted as ellipses or circles with the event name and code.

FLIGHT PROCESS. The flight process ($\Pi$) is a time-history of one or several flight variables, which characterize a certain aspect of the system behavior. Flight processes are used to model dynamic properties of the vehicle, flight control tactics including human piloting and pilot errors, functions and malfunctions of onboard systems, and weather conditions. Every process has its own purpose in the cause-and-effect structure of flight.

Flight processes can be organized by their nature and purpose in the following groups: vehicle dynamics (**D**), flight control processes (**T**, **O**, **P**), airborne systems functioning and failures (**B**, **F**), external operational conditions (**R**, **W**, **Y**, ...), and other. Flight processes are continuous components of the situational model. They are depicted as arrows marked with process main attributes (type, name, and code).

These phrases characterize various flight processes: $\mathbf{D}_1$: *"roll motion"*, $\mathbf{W}_1$: *"windshear 10 ft/s per 30 ft of altitude"*, $\mathbf{F}_7$: *"engine #1 failed"*, $\mathbf{B}_1$: *"autopilot mode #5"*, $\mathbf{P}_{11}$: *"flaps down from 0$^o$ to 15$^o$"*, $\mathbf{T}_9$: *"turn at 20$^o$ bank and zero sideslip"*, $\mathbf{R}_1$: *"heavy rain, intensity 200 mm/h"*, $\mathbf{Y}_1$: *"wet runway, adhesion factor 0.3*".

The following three types of flight process are used in the human pilot model: piloting tasks, flight state observers, and control procedures.

PILOTING TASK. The piloting task (**T**), or the task, is a manual flight control process. It is carried out using airplane's primary controls (elevator, ailerons, rudder, etc.). Piloting tasks represent flight control with feedback. Every piloting task requires observation of the current flight state modeled by 'state observers' (see below). Examples of piloting tasks are as follows: $\mathbf{T}_4$: *"keep to the centerline during groundroll"*, $\mathbf{T}_5$: *"make coordinated turn at bank +15$^o$"*, $\mathbf{T}_8$: *"keep pitch at 10$^o$ and zero bank during initial climb after liftoff"*.

'STATE OBSERVER'. The flight 'state observer' (**O**) is the process of evaluation of current flight states and comparison of these states with relevant tactical objective (goal state). The aim is to detect an error between these two states sufficient to change the performance of a relevant piloting task. For example, the piloting task $\mathbf{T}_8$ listed above is provided with a state 'observer' $\mathbf{O}_1$ to monitor the vehicle motion in pitch. This 'state observer' may include elementary 'observers' [8] for monitoring the airplane pitch angle, pitch rate and pitch acceleration.

CONTROL PROCEDURE. The use of secondary controls (flaps, spoilers, etc.), as well as single movements with the primary controls, are described by the process type called control procedure (**P**). For example, $\mathbf{P}_1$: *"wheels – up"*, $\mathbf{P}_2$: *"unstick"*, $\mathbf{P}_3$: "flap $0^o \rightarrow 35^o$", $\mathbf{P}_6$: *"throttles - to idling"*.

ELEMENTARY FLIGHT SITUATION. The elementary flight situation (**s**) is a primary cause-and-effect relationship between two events and one or several homogeneous processes, i.e.: $\mathbf{s} = (\mathbf{E}_i, \mathbf{E}_k, \{\Pi_1, ..., \Pi_{N(\Pi)}\})$.

It begins at the source event $\mathbf{E}_i$ and ends at the target event $\mathbf{E}_k$, incorporating a set of processes $\Pi_j$ running between $\mathbf{E}_i$ and $\mathbf{E}_k$. The event $\mathbf{E}_i$ ($\mathbf{E}_k$) is called the opening (closing) event for $\Pi_j$ as it triggers the process $\Pi_j$ on (off).

FLIGHT SCENARIO. Basically, the flight scenario (**S**) is a plan of a flight situation. It formalizes the content and the logic of the situation including flight control and operational conditions. Any flight scenario **S** is formed of two sets of objects - flight events, $\Omega(\mathbf{E})$, and flight processes, $\Omega(\Pi)$. They represent, respectively, the discrete and continuous components of the flight situation model.

Examples are as follows: $\mathbf{S}_1$: *"Normal takeoff"*, $\mathbf{S}_3$: *"Aborted takeoff with left engine out"*, $\mathbf{S}_{12}$: *"Groundroll on wet runway"*, $\mathbf{S}_7$: *"Takeoff with two right hand engines out"*, $\mathbf{S}_{10}$: *"Stall in takeoff configuration"*, $\mathbf{S}_{19}$: *"Cruise mode at 450 kn and 30,000 ft"*.

A flight scenario is depicted as a directed graph with the flight events shown as vertices and the flight processes as arcs.

EXAMPLE. The ‘event-process’ formal language can be used to plan various flight cases, both actual and hypothetical, for autonomous simulation. An example of such a scenario for high speed transport airplane is shown in **Fig. 2**. This is a complex flight situation **S**: *“Takeoff with engine #1 out under crosswind and wet runway conditions”*. The diagram depicts main cause-and-effect relationships between the piloting tactics and several internal and external operational factors during the groundroll and takeoff phases of flight.

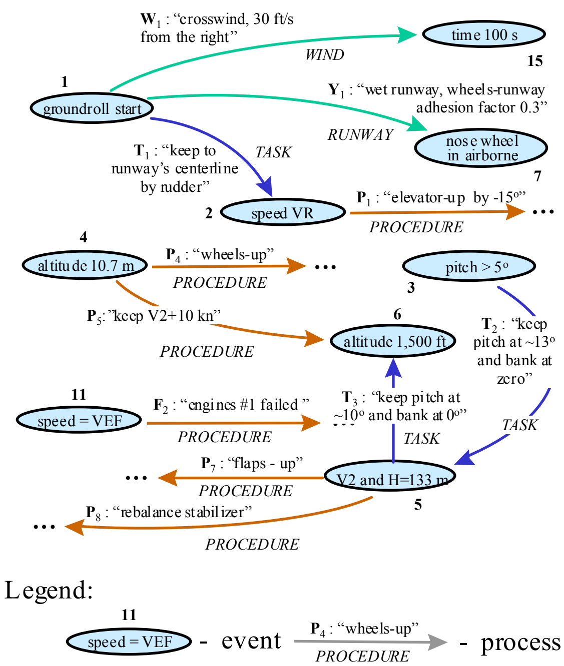


**Fig. 2**. Complex flight scenario **S**: “Takeoff with engine # 1 out under crosswind and wet runway conditions”

Following is a brief description of this scenario.

1. The flight event calendar of **S**, Ω(**E**), includes nine events: $\mathbf{E}_1$: *“groundroll start”*, $\mathbf{E}_2$: *“speed VR”*, $\mathbf{E}_{11}$: *“speed VEF”*, $\mathbf{E}_7$: *“nose wheel in airborne”*, $\mathbf{E}_3$: *“pitch > 5°”*, $\mathbf{E}_4$: *“H = 10.7 m”*, $\mathbf{E}_5$: *“V2 and H = 133 m”*, $\mathbf{E}_6$: *“altitude 1,500 ft”*, and $\mathbf{E}_{15}$: *“time 100 s”*.

2. The list Ω(Π) consists of eleven processes attached to the discrete framework Ω(**E**). Manual control processes are three piloting tasks with feedback ($\mathbf{T}_1$, $\mathbf{T}_2$, and $\mathbf{T}_3$) and five control procedures ($\mathbf{P}_1$, $\mathbf{P}_4$, $\mathbf{P}_5$, $\mathbf{P}_7$, and $\mathbf{P}_8$). Also modeled is one internal operational factor (engine #1 failure, $\mathbf{F}_2$, which is considered as an artificial control procedure), and two external operational factors-processes (wet runway $\mathbf{Y}_1$ and crosswind $\mathbf{W}_1$).

3. As an example, a combination of demanding operational conditions on the groundroll is arranged as follows. Beginning from the event $\mathbf{E}_1$: *“groundroll start”*, two operational factors are applied, $\mathbf{Y}_1$: *“wet runway, wheels-runway adhesion factor 0.3”* and $\mathbf{W}_1$: *“crosswind, 30 ft/s from the right”*. Also, from $\mathbf{E}_1$, the model commences the piloting task $\mathbf{T}_1$: *“keep to runway’s centerline by rudder”*.

4. When the rotation speed is achieved (at $\mathbf{E}_2$: *“speed VR”*), the pilot applies a control action $\mathbf{P}_1$: *“elevator-up by -15°”* to rotate the airplane. (Note that the process $\mathbf{Y}_1$ terminates at the event $\mathbf{E}_7$: *“nose wheel in airborne”*.) When the airplane reaches the pitch attitude about 5° ($\mathbf{E}_3$: *“pitch > 5°”*), the pilot will be maintaining the pitch attitude at about 13° for initial climb (the task $\mathbf{T}_2$: *“keep pitch at ~13°”*).

5. At the engine failure event ($\mathbf{E}_{11}$: *“speed VEF”*), a mechanical failure is added to the scenario through the process $\mathbf{F}_2$: *“engine #1 failed”*. (For continued takeoff VEF is selected between V1 and VR to test the worst possible cases).

6. At the altitude of about 30 ft ($\mathbf{E}_4$: *“altitude 10.7 m”*) the undercarriage retraction process is initiated, $\mathbf{P}_4$: *“wheels-up”*. Simultaneously, the pilot is attempting to maintain the indicated airspeed above the V2 level ($\mathbf{P}_5$: *”keep V2+10 kn”*).

7. Once V2 has been established and the altitude is close to 400 ft (the event $\mathbf{E}_5$: *“V2 and H=133 m”*), the control tactics in pitch is modified by introducing a new piloting task $\mathbf{T}_3$: *“keep pitch at ~10° and bank at 0°”*. Note that the command pitch may be adjusted further to keep IAS slightly above the V2 level if possible. Simultaneously (at $\mathbf{E}_5$), two control procedures begin to change the airplane configuration for further climb, namely: $\mathbf{P}_7$: *“flaps - up”* and $\mathbf{P}_8$: *“rebalance [horizontal] stabilizer”*. The examined takeoff scenario ends at the event $\mathbf{E}_6$: *“altitude 1,500 ft”* or $\mathbf{E}_{15}$: *“time 100 s”*, whichever comes first.

Thus, though this is a very complex flight situation, nevertheless it can be coded and modeled using only a few formal objects of two types – flight event and flight process. Also, the cause-and-effect structure of such a flight scenario is clear to the pilot.

ALGORITHM. At any time instant during simulation all the flight events from Ω(**E**) and the flight processes

from $\Omega(\Pi)$ can be grouped according to their current state in the following subsets:
$\Omega(\mathbf{E}) = \Omega^{NR}(\mathbf{E}) \cup \Omega^{JR}(\mathbf{E}) \cup \Omega^{F}(\mathbf{E}) \cup \Omega^{P}(\mathbf{E})$,
$\Omega(\Pi) = \Omega^{NO}(\Pi) \cup \Omega^{O}(\Pi) \cup \Omega^{F}(\Pi) \cup \Omega^{CL}(\Pi)$.

Note that $\Omega^{JR}(\mathbf{E}) \cup \Omega^{F}(\mathbf{E}) = \Omega^{A}(\mathbf{E})$ and $\Omega^{O}(\Pi) \cup \Omega^{F}(\Pi) = \Omega^{A}(\Pi)$.

A formal relationship for executing a flight scenario in simulation can be written as follows:
$(\forall\mathbf{S})(\mathbf{S} = \Omega(\mathbf{E}) \cup \Omega(\Pi))$, $\mathbf{s} = (\mathbf{E}_i, \mathbf{E}_k, \Pi_j)$ $(((\mathbf{E}_i \in \Omega^{P}(\mathbf{E}) \wedge \mathbf{E}_k \notin \Omega^{P}(\mathbf{E}) \wedge \Pi_j \notin \Omega^{CL}(\Pi)) \wedge (t \geq t\,[\mathbf{E}_i \in \Omega^{P}(\mathbf{E})] + \tau)) \Rightarrow \Pi_j \in \Omega^{A}(\Pi)) \vee ((\mathbf{E}_k \in \Omega^{P}(\mathbf{E}) \Rightarrow \Pi_j \in \Omega^{CL}(\Pi))$.

This relationship, together with the algorithms, which implement models of flight events and flight processes, constitute a computational algorithm of the autonomous flight situation model [12].

## TECHNICAL CHARACTERISTICS

Technical characteristics of a software package which implements the autonomous flight situation model are summarized in **Table 1**.

**Table 1**. Some technical characteristics of the autonomous flight situation modeling software

| | |
|---|---|
| Modeled flight situations and their complexity | majority of operational, test and certification flight cases[4] |
| Flight situation planning method | loadable scenario in the form input data files |
| Time to develop a flight scenario ‘from scratch‘ | 5-15 min |
| Number of differential equations | 13-32 |
| Numerical integration techniques | 4th order fixed-step predictor-correctors, 2nd order variable-step Euler method |
| Integration step | 0.01-0.1 sec |
| Programming language | FORTRAN |
| Simulation speed (on a 200 MHz PC) | 40:1 (airborne modes), 5-10:1 (groundroll modes) |
| RAM requirements | ~520 Kbytes |
| Memory required to retain a scenario on disk | 20 Kbytes |
| Number of input characteristics | 20-80 (old aircraft types), 120-350 (new aircraft types) |
| Number of output flight variables | 200-500 |
| Previous applications | 18 aircraft types, including airplanes, helicopters and a tilt rotorcraft; 30+ problems solved; 200+ scenario types |

[4] within the domain covered by the aircraft input characteristics

## CONCLUSION

A generic flight situation model is proposed for engineering simulations of the behavior of the "pilot - airplane - undercarriage - operational conditions' system for a high speed transport airplane. In particular, the model is capable of accounting for undercarriage-runway interactions and human pilot decision making in multi-factor flight situations. Using this model, complex behaviors of the system can be examined on a computer (PC) beginning from the earlier phases of the airplane's life cycle. Piloting and programming skills are not mandatory for the user.

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## NOMENCLATURE

### SYMBOLS

| | |
|---|---|
| $\sqrt{}$ | square root |
| $\forall$ | "each", "every" |
| $\equiv$ | logical equivalence |
| $\Rightarrow$ | logical implication |
| $\rightarrow$ | state transition |
| $\mathbf{\Pi}$ | flight process |
| $\Psi$ | heading angle |
| $\Omega(\mathbf{\Pi})$ | united list of flight processes |
| $\Omega(\mathbf{E})$ | calendar of flight events |
| $\wedge$, **&** | logical 'and' |
| a | linear acceleration |
| **a** | acceleration vector |
| $a_{sound}$ | local speed of sound |
| $A_x$ | C.G. x coordinate |
| $A_y$ | C.G. y coordinate |
| $A_z$ | C.G. z coordinate |
| **B** | 'onboard system function' type process |
| $c\alpha$, $s\alpha$ | interim variables |
| $c\beta$, $s\beta$ | interim variables |
| $c\phi$, $s\phi$ | interim variables |
| $c\theta$, $s\theta$ | interim variables |
| $c\psi$, $s\psi$ | interim variables |
| **D** | direction cosine matrix |
| **D** | 'flight dynamics' type process |
| DCM | direction cosine matrix |
| E | east coordinate in N-E-D axes |
| **E** | flight event |
| **f** | integrated function vector, $\mathbf{f} = (\mathbf{f}_1, \ldots, \mathbf{f}_{13}) \equiv (U, V, W, p, q, r, \varepsilon_0, \varepsilon_1, \varepsilon_2, \varepsilon_3, N, E, H)$ |
| F | force vector |
| **F** | dynamics vector |
| **F** | 'onboard system failure' type process |
| fuel | refers to fuel |
| G | airplane weight |
| g | acceleration due to gravity |
| H | altitude in N-E-D axes (opposite to the 'down' coordinate) |
| i | index; first index |
| **I** | origin inertia matrix |
| IAS | indicated airspeed |
| $\mathbf{I}_{\mathbf{CG}}$ | C.G. inertia matrix |
| *invert* | matrix inversion operation |
| $I_x$ | C.G. moment of inertia in pitch |
| $I_{xz}$ | C.G. product of inertia about x and z axes |
| $I_y$ | C.G. moment of inertia in pitch |
| $I_z$ | C.G. moment of inertia in pitch |
| j | second index |
| J | interim variable |
| m | airplane mass |
| M | moment vector |
| **M** | mass matrix |
| *M* | Mach number |
| *max* | maximum operation sign |
| *min* | minimum operation sign |
| $M_{\dot{w}}$ | pitching moment due to rate of change of normal velocity [11] |
| $m_x$, $m_y$, $m_z$ | interim variables |
| N | north coordinate in N-E-D axes |
| n | load factor |
| **O** | 'state observer' type process |
| p | roll rate |
| **P** | 'control procedure' type process |
| $P_{atm}$ | atmospheric air pressure |
| q | pitch rate |
| r | yaw rate |
| **R** | 'rain' type process |
| **s** | elementary flight situation |
| **S** | flight situation, or flight [situation] scenario |
| t | time |
| **T** | 'piloting task' type process |
| $T_{atm}$ | atmospheric air temperature |
| U | total axial velocity |
| $U_w$ | x component of wind velocity |
| V | total lateral velocity |
| *V* | airspeed |
| $\dot{V}$ | airspeed time derivative |
| $\dot{V}_{wind}$ | wind velocity derivative |
| V1 | takeoff decision speed |
| V2 | speed V2 |
| $V_{down}$ | down velocity |
| $V_{east}$ | east velocity |
| VEF | engine failure introduction speed |
| $V_{ground}$ | airplane speed along ground path |
| $V_k$ | airplane total velocity along the flightpath |
| $V_{north}$ | north velocity |
| VR | rotation speed |
| $V_w$ | y component of wind velocity |
| $V_{wind}$ | wind velocity |
| W | total normal velocity |
| **W** | 'wind' type process |
| $W_w$ | z component of wind velocity |
| x | longitudinal coordinate in axis system |
| y | lateral coordinate in axis system |
| **Y** | 'runway surface condition' type process |
| z | normal coordinate in axis system |
| $Z_{\dot{w}}$ | normal force due to rate of change of normal velocity [11] |
| $\zeta$ | rudder angle |
| $\Sigma$ | summation sign |
| $\alpha$ | fuselage angle of attack |
| $\alpha_{wing}$ | wing angle of attack |
| $\beta$ | sideslip angle |
| $\delta$ | control angle |
| $\varepsilon_0$ | first Euler's parameter |
| $\varepsilon_1$ | second Euler's parameter |
| $\varepsilon_2$ | third Euler's parameter |
| $\varepsilon_3$ | forth Euler's parameter |
| $\phi$ | bank (roll) angle |
| $\gamma$ | flight path angle |
| $\eta$ | elevator angle |
| $\varphi_{wing}$ | wing incidence angle |
| $\theta$ | pitch angle |
| $\rho_{atm}$ | atmospheric air density |
| $\tau$ | time delay |
| $\xi$ | aileron angle |
| $\psi$ | yaw angle |

**SUBSCRIPTS**

| | |
|---|---|
| 1, 2, … | vector component index; engine number |
| 11, 12, … | matrix element index |
| a | aerodynamic |
| atm | atmospheric |
| b | body axes |
| b←s | from stability to body axes |
| br | brake control |
| C.G. | center of gravity |
| d | dynamic |
| e | earth axes |
| e←b | from body to earth axes |
| fl | flap |
| g | gravity |
| i | index; first index |
| inn | inner section |
| j | index; second index |
| k | index; flight path |
| lft | left |
| lg | landing gear |
| out | outer section |
| p | thrust |
| r | relative |
| rgt | right |
| s | stability axes |
| sl | slat |
| sp | spoiler |
| thr | throttle lever |
| W | normal velocity |
| w | wind |
| b←w | from wind to body axes |

**SUPERSCRIPTS**

| | |
|---|---|
| \` | derivative due to time (d/dt) |
| A | 'currently active' (event) state |
| A | 'currently active' (process) state |
| CL | 'closed' ,or 'past' (process) state |
| F | 'frozen' (event) state |
| F | 'frozen' (process) state |
| JR | 'just recognized' (event) state |
| NO | 'not open' (process) state |
| NR | 'not recognized' (event) state |
| O | 'open' (process) state |
| P | 'past' or 'recognized' (event) state |

**APPENDIX. FLIGHT DYNAMICS ALGORITHM**

Following is a computational algorithm of a generic flight dynamics model [10-12].

1. Update the flight path speed components in body axes after integration; update the airplane weight and the flight path velocity:
$U = \mathbf{f}_1$, $V = \mathbf{f}_2$, $W = \mathbf{f}_3$; $G = mg$, $V_k = \sqrt{(U^2 + V^2 + W^2)}$;

2. Update the airplane angular velocity components in body axes; update the Euler's parameters; update the airplane position in 'north-east-down' axes:
$p = \mathbf{f}_4$, $q = \mathbf{f}_5$, $r = \mathbf{f}_6$;
$\varepsilon_0 = \mathbf{f}_7$, $\varepsilon_1 = \mathbf{f}_8$, $\varepsilon_2 = \mathbf{f}_9$, $\varepsilon_3 = \mathbf{f}_{10}$;
$N = \mathbf{f}_{11}$, $E = \mathbf{f}_{12}$, $H = \mathbf{f}_{13}$;

3. Update the inertia matrix (moments and products of inertia are calculated about the airplane C.G.):
$(\forall i, j)(i = x, y, z, xz)(j = \text{empty, fuel, payload})(I_i=\Sigma I_{i\,j})$,
$\mathbf{I}_{CG\,11} = I_x$, $\mathbf{I}_{CG\,22} = I_y$, $\mathbf{I}_{CG\,33} = I_z$, $\mathbf{I}_{CG\,13} = I_{xz}$, $\mathbf{I}_{CG\,31} = I_{xz}$,
$(\forall i, j)\ (i, j = 1, \ldots, 6)\ (ij \notin \{11, 22, 33, 13, 31\} \Rightarrow (\mathbf{I}_{CG\,ij} = 0)$;

4. Calculate the origin inertia matrix:
$\mathbf{I}_{11} = \mathbf{I}_{CG\,11} + (A^2_y + A^2_z)\,m$,
$\mathbf{I}_{12} = \mathbf{I}_{CG\,12} + (A_x A_y)\,m$,
$\mathbf{I}_{13} = \mathbf{I}_{CG\,13} + (A_x A_z)\,m$,
$\mathbf{I}_{21} = \mathbf{I}_{CG\,21} + (A_y A_x)\,m$,
$\mathbf{I}_{22} = \mathbf{I}_{CG\,22} + (A^2_x + A^2_z)\,m$,
$\mathbf{I}_{23} = \mathbf{I}_{CG\,23} + (A_y A_z)\,m$,
$\mathbf{I}_{31} = \mathbf{I}_{CG\,31} + (A_z A_x)\,m$,
$\mathbf{I}_{32} = \mathbf{I}_{CG\,32} + (A_z A_y)\,m$,
$\mathbf{I}_{33} = \mathbf{I}_{CG\,33} + (A^2_x + A^2_y)\,m$;

5. Calculate the rate of change of the quaternions due to time:
$\varepsilon`_0 = -0.5\,(\varepsilon_1 p + \varepsilon_2 q + \varepsilon_3 r)$,
$\varepsilon`_1 = 0.5\,(\varepsilon_0 p + \varepsilon_2 r - \varepsilon_3 q)$,
$\varepsilon`_2 = 0.5\,(\varepsilon_0 q - \varepsilon_1 r + \varepsilon_3 p)$,
$\varepsilon`_3 = 0.5\,(\varepsilon_0 r + \varepsilon_1 q - \varepsilon_2 p)$;

6. Form the 'earth from body' DCM:
$\mathbf{D}_{e\leftarrow b\,11} = \varepsilon^2_0 + \varepsilon^2_1 - \varepsilon^2_2 - \varepsilon^2_3$,
$\mathbf{D}_{e\leftarrow b\,12} = 2\,(\varepsilon_1 \varepsilon_2 - \varepsilon_0 \varepsilon_3)$,
$\mathbf{D}_{e\leftarrow b\,13} = 2\,(\varepsilon_0 \varepsilon_2 + \varepsilon_1 \varepsilon_3)$,
$\mathbf{D}_{e\leftarrow b\,21} = 2\,(\varepsilon_0 \varepsilon_3 + \varepsilon_1 \varepsilon_2)$,
$\mathbf{D}_{e\leftarrow b\,22} = \varepsilon^2_0 - \varepsilon^2_1 + \varepsilon^2_2 - \varepsilon^2_3$,
$\mathbf{D}_{e\leftarrow b\,23} = 2\,(\varepsilon_2 \varepsilon_3 - \varepsilon_0 \varepsilon_1)$,
$\mathbf{D}_{e\leftarrow b\,31} = 2\,(\varepsilon_1 \varepsilon_3 - \varepsilon_0 \varepsilon_2)$,
$\mathbf{D}_{e\leftarrow b\,32} = 2\,(\varepsilon_0 \varepsilon_1 + \varepsilon_2 \varepsilon_3)$,
$\mathbf{D}_{e\leftarrow b\,33} = \varepsilon^2_0 - \varepsilon^2_1 - \varepsilon^2_2 + \varepsilon^2_3$;

7. Obtain the aircraft velocities (flight path speed components) in earth axes:
$V_{north} = U\,\mathbf{D}_{e\leftarrow b\,11} + V\,\mathbf{D}_{e\leftarrow b\,12} + W\,\mathbf{D}_{e\leftarrow b\,13}$,
$V_{east} = U\,\mathbf{D}_{e\leftarrow b\,21} + V\,\mathbf{D}_{e\leftarrow b\,22} + W\,\mathbf{D}_{e\leftarrow b\,23}$,
$V_{down} = U\,\mathbf{D}_{e\leftarrow b\,31} + V\,\mathbf{D}_{e\leftarrow b\,32} + W\,\mathbf{D}_{e\leftarrow b\,33}$;

8. Calculate the vehicle ground speed, the flight path angle and the heading angle:
$V_{ground} = \sqrt{(V^2_{north} + V^2_{east})}$; $\gamma = arctg\,(-V_{down} / V_{ground})$;
$\Psi = arctg\,(V_{east} / V_{north})$;

9. Calculate the Euler's angles through quaternions:
$(\mathbf{D}_{e\leftarrow b\,32} = 0) \Rightarrow (\phi = 0)$,

$(\mathbf{D}_{e\leftarrow b\,32} \neq 0) \Rightarrow (\phi = arctg\,(\mathbf{D}_{e\leftarrow b\,32} / \mathbf{D}_{e\leftarrow b\,33}))$,
$\theta = arcsin\,(max\,(-1, min\,(1, -\mathbf{D}_{e\leftarrow b\,31})))$,
$(\mathbf{D}_{e\leftarrow b\,21} = 0) \Rightarrow (\psi = 0)$,
$(\mathbf{D}_{e\leftarrow b\,21} \neq 0) \Rightarrow (\psi = arctg\,(\mathbf{D}_{e\leftarrow b\,21} / \mathbf{D}_{e\leftarrow b\,11}))$;

10. Calculate some trigonometric functions of the Euler's angles:
$c\phi = cos\,(\phi)$, $s\phi = sin\,(\phi)$, $s\psi = sin\,(\psi)$, $c\psi = cos\,(\psi)$, $s\theta = sin\,(\theta)$, $c\theta = cos\,(\theta)$;

11. Calculate the force and moment components of the 'gravity vector':
$F_{x\,g} = G\,\mathbf{D}_{e\leftarrow b\,31}$, $F_{y\,g} = G\,\mathbf{D}_{e\leftarrow b\,32}$, $F_{z\,g} = G\,\mathbf{D}_{e\leftarrow b\,33}$;
$M_{x\,g} = G\,A_y\,\mathbf{D}_{e\leftarrow b\,33} - G\,A_z\,\mathbf{D}_{e\leftarrow b\,32}$,
$M_{y\,g} = G\,A_z\,\mathbf{D}_{e\leftarrow b\,31} - G\,A_x\,\mathbf{D}_{e\leftarrow b\,33}$,
$M_{z\,g} = G\,A_x\,\mathbf{D}_{e\leftarrow b\,32} - G\,A_y\,\mathbf{D}_{e\leftarrow b\,31}$;

12. Calculate interim variables and complexes:
$m_x = m$, $m_y = m$, $m_z = m - Z_{w`}$,
$J_x = \mathbf{I}_{11}$, $J_y = \mathbf{I}_{22}$, $J_z = \mathbf{I}_{33}$, $J_{xy} = \mathbf{I}_{12}$, $J_{zx} = \mathbf{I}_{13}$, $J_{yz} = \mathbf{I}_{23}$;

13. Calculate the components of the 'mass matrix' below the diagonal:
$(\forall i, j)\ (i, j = 1, \ldots, 6) \Rightarrow (\mathbf{M}_{ij} = 0)$,
$\mathbf{M}_{11} = m_x$, $\mathbf{M}_{15} = m\,A_z$, $\mathbf{M}_{16} = -m\,A_y$,
$\mathbf{M}_{22} = m_y$, $\mathbf{M}_{24} = -m\,A_z$, $\mathbf{M}_{26} = m\,A_x$,
$\mathbf{M}_{33} = m_z$, $\mathbf{M}_{34} = m\,A_y$, $\mathbf{M}_{35} = -m\,A_x - M`_w$,
$\mathbf{M}_{44} = \mathbf{I}_{11}$, $\mathbf{M}_{45} = -\mathbf{I}_{12}$, $\mathbf{M}_{46} = -\mathbf{I}_{13}$,
$\mathbf{M}_{55} = \mathbf{I}_{22}$, $\mathbf{M}_{56} = -\mathbf{I}_{23}$, $\mathbf{M}_{66} = \mathbf{I}_{33}$;

14. Calculate the components of the 'mass matrix' above the diagonal:
$(\forall i, j)\ (i, j = 1, \ldots, 6)\ (j > i) \Rightarrow (\mathbf{M}_{ij} = \mathbf{M}_{ji})$

15. Invert the 'mass matrix' (the INVERT algorithm [10]):
$\mathbf{M}^{-1} = invert\,(\mathbf{M}, 6)$;

16. Calculate the dynamic vector (force components):
$F_{x\,d} = -m_z\,Wq + m_y\,Vr + m(A_x\,(q^2 + r^2) - A_y\,pq - A_z\,rp)$,
$F_{y\,d} = -m_x\,Ur + m_z\,Wp + m(-A_x\,pq + A_y\,(p^2+r^2) - A_z\,rq)$,
$F_{z\,d} = -m_y\,Vp + m_x\,Uq + m(-A_x\,rp - A_y\,rq + A_z\,(q^2+p^2))$;

17. Calculate the dynamic vector (moment components):
$M_{x\,d} = -(J_z - J_y)\,rq + J_{yz}\,(q^2 - r^2) + J_{zx}\,pq - J_{xy}\,pr + m\,(-A_y\,(Vp - Uq) + A_z\,(Ur - W\,p))$,
$M_{y\,d} = -(J_x - J_z)\,pr - J_{yz}\,pq + J_{zx}\,(r^2 - p^2) + J_{xy}\,qr + m\,(A_x\,(Vp - Uq) - A_z\,(Wq - Vr))$,
$M_{z\,d} = -(J_y - J_x)\,qp + J_{yz}\,p\,r - J_{zx}\,qr + J_{xy}\,(p^2 - q^2) + m\,(-A_x\,(Ur - Wp) + A_y\,(Wq - Vr))$;

18. Obtain wind components and their time derivatives in earth axes (the WIND algorithm); calculate the wind speed and its derivative:
$U_{w\,e} = U_{w\,e}(t)$, $W_{w\,e} = W_{w\,e}(t)$, $V_{w\,e} = V_{w\,e}(t)$;
$U`_{w\,e} = U`_{w\,e}(t)$, $W`_{w\,e} = W`_{w\,e}(t)$, $V`_{w\,e} = V`_{w\,e}(t)$;
$V_{wind} = \sqrt{}\,(U^2_{w\,e} + W^2_{w\,e} + V^2_{w\,e})$,
$V`_{wind} = \sqrt{}\,(U`^2_{w\,e} + W`^2_{w\,e} + V`^2_{w\,e})$;

19. Transform the wind components from earth to body axes:
$U_{w\,b} = U_{w\,e}\,\mathbf{D}_{e\leftarrow b\,11} + V_{w\,e}\,\mathbf{D}_{e\leftarrow b\,21} + W_{w\,e}\,\mathbf{D}_{e\leftarrow b\,31}$,
$V_{w\,b} = U_{w\,e}\,\mathbf{D}_{e\leftarrow b\,12} + V_{w\,e}\,\mathbf{D}_{e\leftarrow b\,22} + W_{w\,e}\,\mathbf{D}_{e\leftarrow b\,32}$,
$W_{w\,b} = U_{w\,e}\,\mathbf{D}_{e\leftarrow b\,13} + V_{w\,e}\,\mathbf{D}_{e\leftarrow b\,23} + W_{w\,e}\,\mathbf{D}_{e\leftarrow b\,33}$;

20. Calculate components and magnitudes of the airspeed and the airspeed rate of change:
$U_r = U - U_{w\,b}$, $V_r = V - V_{w\,b}$, $W_r = W - W_{w\,b}$,
$V = \sqrt{}\,(U^2_r + V^2_r + W^2_r)$;
$U`_r = U` - U`_{w\,b}$, $V`_r = V` - V`_{w\,b}$, $W`_r = W` - W`_{w\,b}$,
$V` = \sqrt{}\,(U`^2_r + V`^2_r + W`^2_r)$;

21. Obtain the current atmospheric state parameters from the ATMOSPHERE algorithm; update the Mach number:
$P_{atm}$, $T_{atm}$, $\rho_{atm}$, $a_{sound}$; $M = V / a_{sound}$;

22. Calculate the angles of attack, the sideslip angle, and their trigonometric functions:
$\alpha = arctg\,(W_r / U_r)$, $\alpha_{wing} = \alpha + \varphi_{wing}$,
$c\alpha = cos\,(\alpha)$, $s\alpha = sin\,(\alpha)$, $\beta = arctg\,(V_r / U_r)$,
$c\beta = cos\,(\beta)$, $s\beta = sin\,(\beta)$;
$\alpha` = (U`_r\,W_r - U_r\,W`_r) / (U^2_r + W^2_r)$,
$\beta` = (V`_r\,U_r - V_r\,U`_r) / (U^2_r + V^2_r)$;

23. Form the 'wind to body axes' DCM:
$\mathbf{D}_{w\to b\,11} = c\alpha\,c\beta$, $\mathbf{D}_{w\to b\,12} = -c\alpha\,s\beta$, $\mathbf{D}_{w\to b\,13} = -s\alpha$,
$\mathbf{D}_{w\to b\,21} = s\beta$, $\mathbf{D}_{w\to b\,22} = c\beta$, $\mathbf{D}_{w\to b\,23} = 0$,
$\mathbf{D}_{w\to b\,31} = s\alpha\,c\beta$, $\mathbf{D}_{w\to b\,32} = -s\alpha\,s\beta$, $\mathbf{D}_{w\to b\,33} = c\alpha$;

24. Form the 'stability to body axes' DCM:
$\mathbf{D}_{b\leftarrow s\,11} = c\alpha$, $\mathbf{D}_{b\leftarrow s\,12} = 0$, $\mathbf{D}_{b\leftarrow s\,13} = -s\alpha$;
$\mathbf{D}_{b\leftarrow s\,21} = 0$, $\mathbf{D}_{b\leftarrow s\,22} = 1$, $\mathbf{D}_{b\leftarrow s\,23} = 0$;
$\mathbf{D}_{b\leftarrow s\,31} = s\alpha$, $\mathbf{D}_{b\leftarrow s\,32} = 0$, $\mathbf{D}_{b\leftarrow s\,33} = c\alpha$;

25. Convert the airplane linear and angular velocities from body to stability axes:
$U_s = U_r\,\mathbf{D}_{b\leftarrow s\,11} + W_r\,\mathbf{D}_{b\leftarrow s\,31}$, $V_s = V_r\,\mathbf{D}_{b\leftarrow s\,22}$,
$W_s = U_r\,\mathbf{D}_{b\leftarrow s\,13} + W_r\,\mathbf{D}_{b\leftarrow s\,33}$; $p_s = p\,\mathbf{D}_{b\leftarrow s\,11} + r\,\mathbf{D}_{b\leftarrow s\,31}$,
$q_s = q\,\mathbf{D}_{b\leftarrow s\,22}$, $r_s = p\,\mathbf{D}_{b\leftarrow s\,13} + r\,\mathbf{D}_{b\leftarrow s\,33}$;

26. Obtain inputs from the human pilot model (the algorithms TASKS and PROCEDURES):
$\delta_{fl}(t)$, $\delta_{sl}(t)$, $\delta_{lg}(t)$, $\delta_{sp.inn}(t)$, $\delta_{sp.out}(t)$, $\delta_{br.lft}(t)$, $\delta_{br.rgt}(t)$, $\delta_{thr.1}(t)$, $\delta_{thr.2}(t)$, $\delta_{thr.3}(t)$, $\delta_{thr.4}(t)$; $\eta(t)$, $\xi(t)$, $\zeta(t)$;

27. Calculate the effect of rain and/or ice on the airplane aerodynamics in body axes (the RAIN and ICE algorithms; note that $F_{y\,rain} = M_{x\,rain} = M_{z\,rain} = 0$ and $F_{y\,ice} = M_{z\,ice} = 0$):
$\{ F_{x\,rain}, F_{y\,rain}, F_{z\,rain}, M_{x\,rain}, M_{y\,rain}, M_{z\,rain} \}$,

$\{ F_{x\,ice}, F_{y\,ice}, F_{z\,ice}, M_{x\,ice}, M_{y\,ice}, M_{z\,ice} \}$;

28. Obtain components of the aerodynamic force and moment in body axes (the AERODYNAMICS algorithm):
$\{ F_{x\,a}, F_{y\,a}, F_{z\,a}, M_{x\,a}, M_{y\,a}, M_{z\,a} \}$;

29. Calculate components of the force and moment due to thrust in body axes (the ENGINE algorithm):
$\{ F_{x\,p}, F_{y\,p}, F_{z\,p}, M_{x\,p}, M_{y\,p}, M_{z\,p} \}$;

30. Calculate components of the undercarriage reaction force and moment in body axes (the UNDERCARRIAGE algorithm – see below):
$\{ F_{x\,lg}, F_{y\,lg}, F_{z\,lg}, M_{x\,lg}, M_{y\,lg}, M_{z\,lg} \}$;

31. Calculate components of the total force and moment vector in body axes (note that (i = x, y, z) and (j = d, g, a, p, lg, ice, rain) are symbolic substitutions):
$(\forall i, j)$ (i = x, y, z) (j = d, g, a, p, lg, ice, rain)
$(F_i = \Sigma F_{i\,j},\ M_i = \Sigma M_{i\,j})$;
$\mathbf{F}_1 = F_x,\ \mathbf{F}_2 = F_y,\ \mathbf{F}_3 = F_z,\ \mathbf{F}_4 = M_x,\ \mathbf{F}_5 = M_y,\ \mathbf{F}_6 = M_z$;

32. Calculate components of the load factor in body axes ((i = x, y) and (j = a, p, lg, ice, rain) are symbolic substitutions):
$(\forall i, j)$ (i = x, y) (j = a, p, lg, ice, rain)
$(n_i = (\Sigma F_{i\,j}) / G),\ n_z = -(\Sigma F_{z\,j}) / G)$;

33. Obtain linear accelerations in body axes:
$(\forall i, j)$ (i =x, y, z) (j = g, a, p, lg, ice, rain)
$(a_i = (\Sigma F_{i\,j}) / m)$;

34. Transform linear accelerations from body to earth axes:
$a_{x\,e} = a_x \mathbf{D}_{e\leftarrow b\,11} + a_y \mathbf{D}_{e\leftarrow b\,12} + a_z \mathbf{D}_{e\leftarrow b\,13}$,
$a_{y\,e} = a_x \mathbf{D}_{e\leftarrow b\,21} + a_y \mathbf{D}_{e\leftarrow b\,22} + a_z \mathbf{D}_{e\leftarrow b\,23}$,
$a_{z\,e} = a_x \mathbf{D}_{e\leftarrow b\,31} + a_y \mathbf{D}_{e\leftarrow b\,32} + a_z \mathbf{D}_{e\leftarrow b\,33}$;

35. Calculate the acceleration vector; form components of the linear and angular accelerations:
$(\forall i, j)$ (i=1, …, 6; $\mathbf{a}_i = 0$; (j=1, …, 6; $\mathbf{a}_i = \mathbf{a}_i + \mathbf{M}^{-1}_{ij} \mathbf{F}_j$));
$U` = \mathbf{a}_1,\ V` = \mathbf{a}_2,\ W` = \mathbf{a}_3;\ p` = \mathbf{a}_4,\ q` = \mathbf{a}_5,\ r` = \mathbf{a}_6$;
$V`_k = \sqrt{(U`^2 + V`^2 + W`^2)}$;

36. Form the derivative vector for integration:
$\mathbf{f}`_1 = U`,\ \mathbf{f}`_2 = V`,\ \mathbf{f}`_3 = W`$;
$\mathbf{f}`_4 = p`,\ \mathbf{f}`_5 = q`,\ \mathbf{f}`_6 = r`$;
$\mathbf{f}`_7 = \varepsilon`_0,\ \mathbf{f}`_8 = \varepsilon`_1,\ \mathbf{f}`_9 = \varepsilon`_2,\ \mathbf{f}`_{10} = \varepsilon`_3$;
$\mathbf{f}`_{11} = V_{north},\ \mathbf{f}`_{12} = V_{east},\ \mathbf{f}`_{13} = -V_{down}$.