# ANALYSIS AND OPTIMIZATION OF HYPERSONIC MANEUVERING OF A TRANSATMOSPHERIC VEHICLE UNDER UNCERTAINTY USING FUZZY TREES

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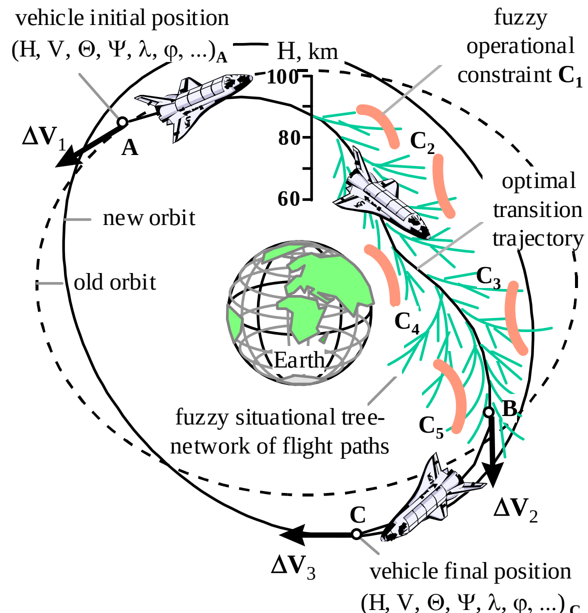
## ABSTRACT

A high-altitude hypersonic maneuver of a transatmospheric vehicle for changing the inclination of its orbital plane is examined using mathematical modeling and fuzzy set theory techniques. Knowledge of the vehicle flight dynamics and control is formalized as an artificial memory structure in the form of a fuzzy situational tree-network (FSTN). The FSTN is comprised of a large number of interrelated (branching) fuzzy flight situations and fuzzy transitions, which may occur under the effect of selected operational factors. In this paper an algorithm developed for FSTN construction and analysis in the presence of multiple fuzzy constraints is introduced. A modified Bellman-Zadeh's method is employed for FSTN-based flight control optimization. Some numerical results of FSTN construction and analysis and FSTN-based flight control optimization and flight modeling are presented. Potential advantages and possible drawbacks of the technique are briefly summarized.

## INTRODUCTION

Safe and affordable transatmospheric flight is a central component of future Earth-orbit, orbit-Earth and orbit-orbit operations [1]. For an advanced aerospace plane it is important to have a capability of changing the inclination of its orbit. One of the possible methods to accomplish this task is to employ the vehicle aerodynamics through a descent-turn-ascent hypersonic maneuver (**Fig. 1**). To commence a descent, a deceleration impulse $\Delta\mathbf{V}_1$ is applied at point **A** in the old orbit**.** Then, a descent-turn-ascent maneuver is performed in the upper atmosphere (65-85 km) by maintaining a required program of changing the vehicle's angles of attack and roll. During the ascending segment of the maneuver the flight altitude is gained at the expense of the vehicle's kinetic energy. Finally, the vehicle speed and the altitude required in a new orbit are restored by applying two accelerating impulses $\Delta\mathbf{V}_2$ and $\Delta\mathbf{V}_3$ at points **B** and **C**, respectively (see **Fig. 1**).




Legend:

$\mathbf{C}_i$ - other fuzzy constraints, i = 2, 3, …
$\Delta\mathbf{V}_1$ - deceleration impulse
$\Delta\mathbf{V}_2$ and $\Delta\mathbf{V}_3$ - acceleration impulses

**Fig. 1**. High-altitude hypersonic maneuver of a transatmospheric vehicle

## PROBLEM

Such a maneuver must conform to several interrelated operational constraints (see **Fig. 1**). These include the constraints on a maximum temperature, dynamic pressure and load factor, as well as limits imposed on the vehicle state and control variables.

Also, there are various uncertainty factors which are to be taken into account in design and operation of this type of vehicle. These include unpredictable fluctuations of air density in the upper atmosphere, uncertainties of the vehicle's aerodynamic characteristics under such conditions, variations in the vehicle operational weight, a limited knowledge of the optimum flight control, pilot errors, and a limited experience in performing such maneuvers in the past.

Thus, appropriate techniques are required to study the vehicle flight dynamics and control under such conditions. The problem under study may be formulated is as follows: how to adequately model the behavior of the "pilot (automatic control system) - transatmospheric vehicle" system under uncertainty conditions in the presence of multiple constraints?

## SOLUTION APPROACH

Artificial intelligence (AI) techniques are employed to address the problem. Knowledge of the vehicle flight dynamics and control under uncertainty is modeled as an artificial memory structure in the form of a fuzzy situational tree-network (FSTN) [4]. The FSTN is comprised of many interrelated (branching) fuzzy flight situations and transitions which may occur under the effect of key operational factors (**Fig. 1**).

The purpose of the FSTN is to thread a complex operational domain of flight in a special, systematic way. Examined are potentially unsafe zones located at the operational constraints, as well as safe flight modes which can be used to bring the vehicle to a goal subset (point **B** - see **Fig. 1**). The FSTN is built using a mathematical model of the vehicle flight dynamics and fuzzy set techniques. Then, a subtree of optimum flight paths, together with the associated control tactics are derived from the FSTN by applying a modified Bellman-Zadeh's method [3]. Finally, flight control is organized as a multi-stage situation based fuzzy decision making process.

## FSTN CONCEPT

In order to account for an uncertain and complex character of the behavior of the "pilot (automatic control system) - transatmospheric vehicle" system, appropriate AI models for representing knowledge of complex operational domains of flight are required. One of such models is called the fuzzy situational tree-network (**Fig. 2**).

PURPOSE. The purpose of the FSTN is to map and accumulate knowledge of a complex (multi-factor) operational domain of flight. Basically, the FSTN is constructed as a result of action of various control inputs and anticipated operational factors (conditions) combined in a special, systematic way.

STRUCTURE. The FSTN consists of a large number of fuzzy situations (nodes), and their relationships - fuzzy transitions (arcs). A cause-and-effect chain of several fuzzy situations and their transitions forms a fuzzy branch, which stands for some flight path option. Note that being placed in any situation-node on the branch, it is possible to make forecasts or recalls of all the future or previous flight paths associated with this particular fuzzy situation.

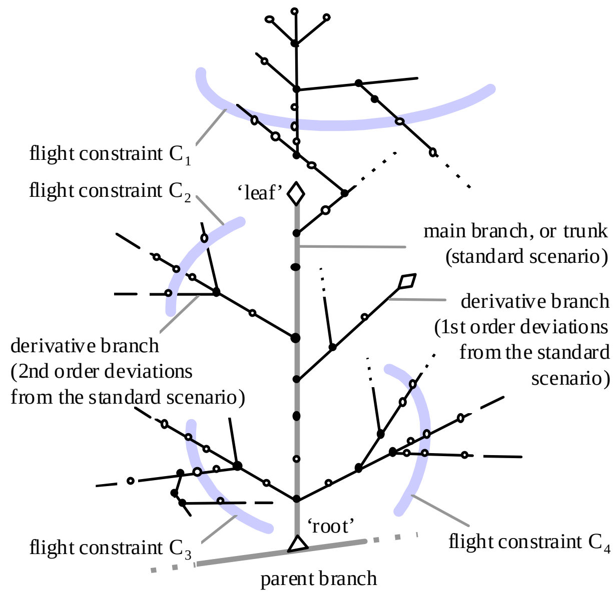



**Fig. 2**. Fuzzy situational tree-network of flight

FUZZY SITUATION. Fuzzy situations in the FSTN may be considered as imprecise 'snapshots', or fuzzy images, of actual flight situations. Each situation is characterized by a fuzzy state vector, a list of key flight events and flight processes (current and recently completed), and a vector of integral characteristics of the situation (e.g.: quality, safety, etc.) [4].

OPERATIONAL FACTORS. Operational factors of flight are considered as some non-standard disturbances or circumstances in the system behavior. They change the standard flight control scenario and as a result affect the normal flight path. The situational flight modeling technique [4] provides a method to account for various heterogeneous operational factors in the FSTN structure.

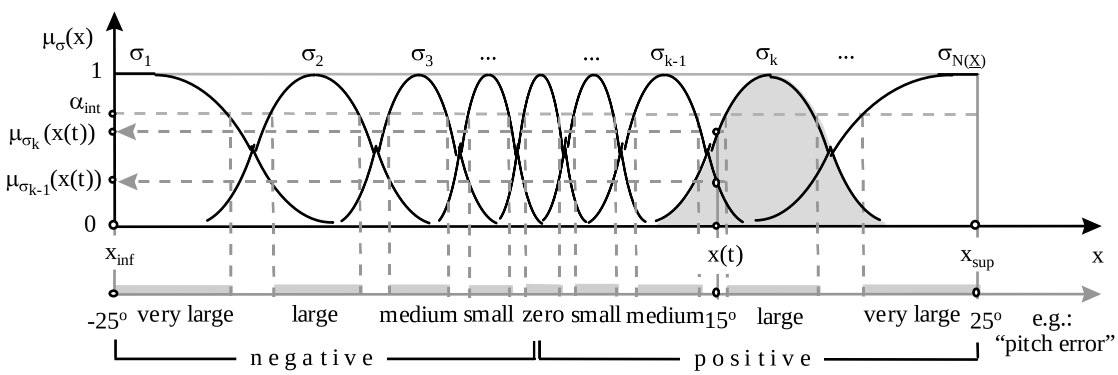
COMPONENTS. The FSTN structure is comprised of the following components or 'building blocks' (see **Fig. 2**): the root situation, the leaf situation, the bud situation, the reference situation. Also, the FSTN has one main branch and a number of derivative branches.

The root situation starts the segment of flight under examination. It links the FSTN (its subtree or a branch) to its 'parent' - a branch representing a previous segment of flight. The leaf situation finishes a branch and denotes a goal situation of a flight control scenario [2, 4]. The main branch (trunk) is constructed according to the main scenario. Derivative (secondary) branches of the first, second or higher order implement modified scenarios.

A reference type situations constitute the majority in the FSTN. During FSTN based flight modeling and control they will be used to monitor the evolution of flight with respect to fuzzy constraints. In addition to this function, bud situations are also used for implanting new branches.

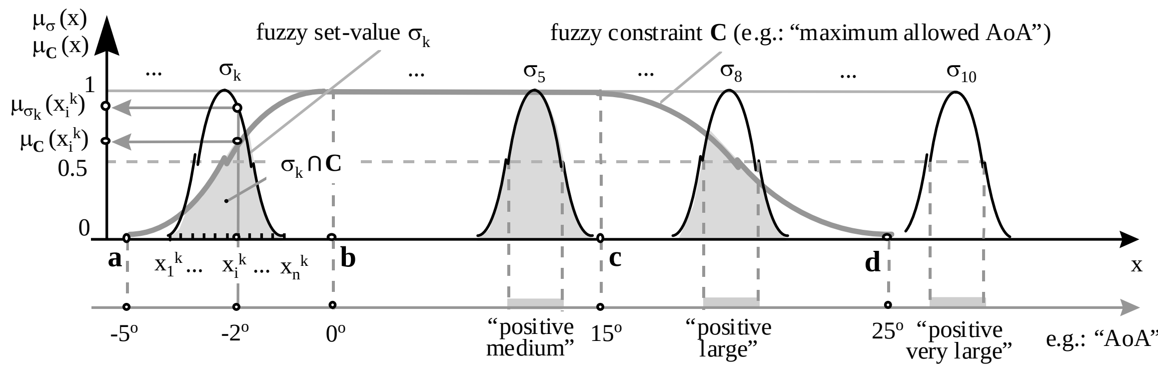
FUZZY STATE. System states are described by linguistic variables that take fuzzy values from fuzzy measurement scales (**Fig. 3**). Fuzzification of the system state space allows to cover a sufficiently large operational domain of flight in the FSTN and thus mitigate, to a certain extent, the effect of a 'curse of dimensionality'.

FUZZY CONSTRAINT. To account for the uncertainty of our knowledge of the vehicle's flight envelope, operational flight constraints are also formalized by fuzzy sets. Within the FSTN structure, a fuzzy constraint may be considered as an external object, or a strip, attached to one or several branches (**Fig. 2**). Note that the position of a fuzzy constraint in the FSTN can be revealed only during FSTN construction. The degree of compatibility of a fuzzy state (situation) and a fuzzy constraint (**Fig. 4**) is measured using the intersection operation for fuzzy sets. These quality measures will be used in flight control optimization.




$(\forall x(t))\ (x(t) \in [x_{inf};\ x_{sup}])\ (\exists \sigma_k)\ (\sigma_k \in \underline{X})\ (\mu_{\sigma_k}(x(t)) = \max \mu_{\sigma_i}(x(t)),\ i=1, ..., N(\underline{X}),\ k \in \{1, ..., N(\underline{X})\}) \Rightarrow \underline{x}(t) \cong \sigma_k)$

**Fig. 3**. Fuzzy measurement scale of a linguistic flight variable




$$\mu_{\mathbf{C}}(\sigma_k) = \frac{\text{card}(\sigma_k \cap \mathbf{C})}{\text{card}(\sigma_k)} \qquad \text{card}(\sigma_k \cap \mathbf{C}) = \sum_{i=1}^{n} \min[\mu_{\sigma_k}(x_i^k),\ \mu_{\mathbf{C}}(x_i^k)] \qquad \text{card}(\sigma_k) = \sum_{i=1}^{n} \mu_{\sigma_k}(x_i^k)$$

**Fig. 4**. Fuzzy constraint and fuzzy values of a linguistic flight variable

CONSTRUCTION PRINCIPLE. The FSTN is constructed for a critical segment of flight (10-100 seconds long). The segment length depends on the vehicle type, system dynamics and a flight mission. The FSTN construction process is based on the autonomous flight modeling technique and a FSTN's genotype [4]. The main branch is formed in flight simulation according to the main scenario. Derivative branches represent scenarios modified due to the FSTN's genotype.

The overall objective of the FSTN construction process is to learn and memorize knowledge of possible fuzzy flight situations and their transitions under the effects of anticipated operational conditions.

## ALGORITHM

The algorithm for FSTN construction, analysis and optimization includes the following main phases:

- specification of the main fuzzy metrics of situational flight space
- development of the main flight scenario
- specification of the FSTN genotype
- FSTN training
- FSTN growth monitoring
- FSTN based optimization of flight control (to achieve a goal set)
- analysis and verification of the FSTN content, and
- visualization and generalization of knowledge retained in the FSTN.

Following is a brief introduction to this algorithm.

1. Specification of the main fuzzy metrics of situational flight space includes: definition of vectors of the system linguistic variables, specification of the fuzzy measurement scales in state and control spaces, and specification of the fuzzy flight constraints selected for monitoring [4].

2. To develop a main flight scenario a calendar of flight events and a list of flight processes are required [4]. These objects will be used to formalize the content and the logic of the operational domain of flight.

3. Specification of the FSTN genotype includes the following tasks: selection of a time increment for quantifying branches (this is a distance between two neighboring situations), specification of operational factors of flight and their levels, and definition of the rules for implanting derivative branches. These rules may be presented, for example, as a system of constraints in the Cartesian product of pairs of the main system spaces (states, controls and factors). In order to define the FSTN genotype, pseudo-physical relationships of flight, which represent fundamental knowledge of aerodynamics, flight dynamics and flight control, may be used as well.

4. FSTN training is a multi-stage process of exploration of fuzzy situational space and implanting derivative branches into the FSTN according to the genotype. The main branch is modeled according to a main (standard) scenario. A bud type situations are defined in the main branch. Then, derivative branches are implanted in those bud situations on the branch where the flight scenario deviates from its standard. Derivative branches are formed and processed exactly in the same manner.

5. FSTN growth is periodically monitored during the FSTN growing process. For this purpose a subset of target fuzzy situations of the transitions generated during each construction step is projected on selected phase planes of the system state space. Evolution of these projections as a function of the FSTN construction step is monitored, and analyses of various sections of the FSTN structure are performed. Through this process, the direction of FSTN branching, its density and other quality characteristics of the FSTN can be controlled.

6. Search and optimization of flight paths within the FSTN are conducted by means of a modified Bellman-Zadeh's technique [3]. First, a goal set of fuzzy flight situations is defined. This set may be composed of two subsets: main goals and alternative goals [4]. Alternative goals are applied when the main one cannot be physically achieved under a given condition. Using a dynamic programming (back propagation) technique, control tactics are optimized for all the situations which are linkable through the FSTN with at least one fuzzy goal, main or alternative.

7. Analysis and verification of the FSTN content are performed after accomplishing the FSTN construction process. The algorithms for FSTN growth monitoring may be used for this purpose as well. Safety (quality) characteristics of all the situations in the FSTN and integral characteristics of the FSTN are calculated. Finally, the FSTN quality is assessed in flight simulation experiments on a computer.

8. FSTN knowledge visualization and generalization accomplish the FSTN construction process. The purpose of visualization depends on FSTN application. It may include various graphical and quantitative mappings of the FSTN topology and knowledge sections for more detailed analyses. The purpose of

knowledge generalization is to derive strategies for decision making at a higher level of control. It may also include the assessment of the overall safety (quality) status of the system and the identification of the most vulnerable constraints and the most critical operational factors, etc.

## EXAMPLE

The algorithms introduced above have been programmed and tested on a computer. This test includes a FSTN based analysis, optimization and modeling of flight of a transatmospheric vehicle during a high-altitude hypersonic maneuver under the conditions of uncertainty [2]. The objective of this experiment is two fold: (1) to study the properties of the FSTN prototype, and (2) to examine the vehicle flight dynamics and control under uncertainty.

The uncertainty factors which affect the system behavior are grouped as follows: imprecise knowledge of the vehicle flight dynamics and control tactics, atmospheric density fluctuations, imprecision of observations of system states, variations of the vehicle's design and control system parameters.

System state and control vectors comprise nine and three components, respectively: $(x^1, ..., x^9) = (H, V, dV/dt, \Theta, d\Theta/dt, \Psi, d\Psi/dt, \lambda, \varphi)$ and $(u^1, u^2, u^3) = (\alpha, \gamma, \Delta\rho)$. The main control variables are the vehicle angles of attack $\alpha$ and roll $\gamma$, which must be maintained during the maneuver. The third control variable is artificial, this is the operational factor $\Delta\rho$. It represents deviations of the atmospheric air density from its standard and is used for FSTN construction together with the two main control variables.

Fuzzy measurement scales of linguistic variables $\underline{x}$ and $\underline{u}$ are specified in **Table 1** (see also **Fig. 3**).

**Table 1**. Fuzzy measurement scales of linguistic variables of flight

| variable | space | unit | range | N($\underline{X}$) | $\chi$ |
|---|---|---|---|---|---|
| H | X | km | [65; 80] | 9 | 1 |
| V | X | km/s | [6.5; 7.8] | 9 | 1 |
| dV/dt | X | m/s$^2$ | [-30; 0] | 9 | 1 |
| $\Theta$ | X | degr | [-3; 3] | 9 | 1.5 |
| d$\Theta$/dt | X | degr/s | [-0.3; 0.3] | 9 | 1 |
| $\Psi$ | X | degr | [-0; 10] | 9 | 1 |
| d$\Psi$/dt | X | degr/s | [0; 0.3] | 9 | 1 |
| $\lambda$ | X | degr | [0; 12] | 9 | 1 |
| $\varphi$ | X | degr | [0; 1.12] | 9 | 1 |
| $\alpha$ | U | degr | [15; 45] | 9 | 1 |
| $\gamma$ | U | degr | [30; 90] | 7 | 1 |
| $\Delta\rho$ | U/$\Phi$ | % | [-30; 30] | 7 | 1 |

The fuzzy constraints selected for monitoring are summarized in **Table 2** (see also **Fig. 4**), together with the initial conditions of flight (a numeric state vector $x_0$, which corresponds to the FSTN's root).

**Table 2**. Specification of fuzzy constraints and initial conditions of flight

| $\underline{x}$/$\underline{u}$/y, unit | a | b | c | d | $x_o$ |
|---|---|---|---|---|---|
| H, km | 60. | 65. | 80. | 85. | 80. |
| V, km/s | 5. | 6.5 | 7.8 | 9. | 7.803 |
| dV/dt, m/s$^2$ | -30. | -25. | -5. | 0. | 0. |
| $\Theta$, degr | -4. | -2. | 2. | 4. | -2. |
| d$\Theta$/dt, degr/s | -0.3 | -0.2 | 0.2 | 0.3 | 0. |
| $\Psi$, degr | -2.0 | 0. | 10. | 12. | -0.06 |
| d$\Psi$/dt, degr/s | 0. | 0.05 | 0.25 | 0.3 | 0. |
| $\lambda$, degr | -2. | 0. | 1.12 | 3.12 | 0. |
| $\varphi$, degr | -2. | 0. | 12. | 14. | 0. |
| $\alpha$, degr | 10. | 20. | 40. | 50. | 30. |
| $\gamma$, degr | 25. | 35. | 85. | 95. | 65. |
| $\Delta\rho$, % | -100. | -90. | 90. | 100. | 0. |
| $n_x$, - | -4. | -3. | 3. | 4. | - |
| $n_z$, - | -4. | -3. | 3. | 4. | - |
| $T_c$, K | -273. | -173. | 2000. | 3000. | - |
| q, N/m$^2$ | 0. | 10. | 3000. | 3500. | - |

The following two parameters are used to monitor and analyze the FSTN growth: the number of examined fuzzy situations, N(**S**), and the number of fuzzy transitions, N(**T**), as a function of the construction step, *i*.

The process of FSTN construction is depicted in **Fig. 5** in Appendix in projection on the phase plane (H, V) of their target situations. The FSTN starts growing in a subdomain with the fuzzy value indices $I(H,V) \in \{(8,9), \ldots, (7,8)\}$ corresponding to a 'large' altitude and 'high' speed. Then, the FSTN expands in volume, moving from right to left on the phase plane, and the process finishes in a fuzzy space region with $I(H) \in \{5, \ldots, 9\}$ and $I(V) \in \{1, 2, 3\}$, i.e. with 'low' speed and 'high' altitude values. The characteristics N(**S**) = f(*i*) and N(**T**) = f(*i*) are depicted in **Fig. 6** in Appendix.

In total, this FSTN prototype has accumulated 201,676 fuzzy transitions. Note that the FSTN has received a significant internal damage to its structure at the step *I* = 5 (due to power supply failure). However, flight simulation experiments demonstrate (see below) that this damage does not affect the FSTN performance in terms of the flight control quality.

**Fig. 7** demonstrates examples of projection of a subtree containing 73,287 fuzzy transitions, which correspond to the fuzzy index I(Δρ)=2 of the air density variable (this hypothesis is labeled as 'Δρ=2'). The projections are made on the planes 'heading angle - flight path angle', 'latitude - longitude' and 'altitude - speed' of the target situations of these transitions.

Similar subtree mappings have been obtained for other density hypotheses.

For flight control optimization a set **G** of main flight goals is specified as follows: **G** = { $\underline{x}_1$, …, $\underline{x}_{N(\mathbf{G})}$ }, where $\underline{x} = (\underline{x}^1, \ldots, \underline{x}^9)$ and $I(\underline{x}^1, \ldots, \underline{x}^9) \in (1,2) \times (8,9) \times (9,9) \times (7,8) \times (5,6) \times (8,9) \times (1,3) \times (5,5) \times (6,6)$. In $I(\underline{x}^1, \ldots, \underline{x}^9)$ a pair $(I(x^k{}_1), I(x^k{}_2))$ of the lower and upper indices of a fuzzy set-value $\underline{x}^k$ specifies a projection of the goal set on the *k*-th component, *k* = 1, … 9.

Several hypotheses of air density variations due to flight altitude were employed during optimization. The dynamics of subsets of source fuzzy situations during optimization for a complex air density hypothesis 'Δρ=4262' is shown in **Fig. 8** in projection on the 'altitude - speed' fuzzy phase plane. Note that the number of optimized fuzzy situations and transitions constitutes a small part of the FSTN.

An example of the optimum fuzzy control policy for the hypothesis 'Δρ=4262' is shown in **Fig. 9**. This optimum subtree has a boomerang-type projection that matches general expectations regarding the shape of a bunch of optimum paths for this particular maneuver.

In order to remedy local damages occurred in the FSTN structure, a special technique for associative processing of neighboring fuzzy situations can be applied during optimization. In this technique, the fuzzy affinity vector [4], $\delta = (\delta^1, \ldots, \delta^9)$, was used to define the size and the structure of a subset of the fuzzy situations which surround an 'empty' (i.e. non-linked) situation. As a result, missing transitions between such situations from the damaged zone can be restored.

Integral characteristics of the FSTN construction process are summarized in **Table 3**.

Note that as a result of training this particular FSTN prototype has accumulated a significant total flight time (AI 'piloting experience') equal to about 672 flight hours. It is important that this knowledge systematically covers various complex flight situations and is presented in a computationally manageable format. Optimum flight paths and control tactics can be dynamically derived from the FSTN to meet specific flight goals under anticipated conditions.

**Table 3**. Some integral characteristics of the FSTN prototype and construction process

| | |
|---|---|
| State vector, x | $(x^1, ..., x^9)$ |
| Control vector, u | $(u^1, u^2)$ |
| Examined external operational factor | Δρ |
| Number of fuzzy constraints | 16 |
| Duration of a fuzzy transition, Δ | 12 s |
| Duration of FSTN construction | ~25 hrs* |
| Number of cycles in FSTN branching | 21 cycles |
| Total number of situations in FSTN | 40,334** |
| Total number of transitions in the FSTN | 201,667** |
| Number of transitions with $\mu_C(\underline{x}(t+\Delta)) > 0.7$ | >170,000 |
| Total number of rejected situations | ~770,000*** |
| Number of transitions violating constraints | ~11,500 |
| Programming language | FORTRAN |
| Size of the FSTN prototype | 17.6 Mb |
| Size of auxiliary (address) tables | 2.0 Mb |
| Code size (FSTN constructor) | 3.9 Mb |
| Number of situations linked with goal set | ~1,500 |
| Average duration of flight maneuver | 100-110 s |
| Memory required to retain knowledge of one second of flight, bytes | 8.46 **** |
| FSTN 'experience' (total flight time) | 672.2 hrs |

Notes:
* on a 166 MHz PC
** within the flight envelope
*** due to genotype mismatch
**** includes FSTN and address tables

In total, 230 flight simulation experiments have been conducted to test the performance of FSTN based fuzzy control. The objective is to study the effects of various uncertainty parameters on the quality of fuzzy control policies. Examples of flight simulation are presented in **Fig. 10-11** in Appendix for two vehicle weights (minimum and maximum) and a complex air density hypothesis 'Δρ = 4264'.

Note that the frequency of accessing knowledge in the control policy during 'decision making' in flight simulation is low (0.1-2 Hz). The control algorithm is simple: system state fuzzification → finding a matching reference situation → retrieval of the appropriate 'fuzzy situation – control decision' pair → control decision defuzzification → control input implementation. However, large memory resources are required to accommodate the FSTN (see **Table 3**).

## CONCLUSION

Simulation experiments demonstrate that the FSTN concept can be implemented on a computer and can be used for studying the vehicle flight dynamics and control under uncertainty conditions. The FSTN can be trained to accumulate knowledge of an anticipated operational domain using a situational model of flight. In this process, however, two issues are important: FSTN growth control and FSTN content validation.

FSTN based fuzzy control is goal-oriented and robust enough against reasonable variations in the examined uncertainty factors. The FSTN has characteristics of ‘experience’ or competence, such as total flight time, and other, which are similar to those ones used to measure the human pilot’s tactical experience. Local damages to the FSTN or missing knowledge can be rectified; these deficiencies do not affect critically the quality of fuzzy flight control.

The frequency of data retrieval from the FSTN is low (within 1-5 Hz). FSTN processing algorithms are also simple. For a realistic application the amount of direct access memory required to accommodate the FSTN is estimated within $10^1$-$10^2$ Gbytes ($10^1$-$10^2$ bytes per one second of flight time).

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## NOMENCLATURE

| | |
|---|---|
| a | characteristic point of a fuzzy set carrier |
| AI | artificial intelligence |
| **A**, **B**, **C** | characteristic points of the maneuver |
| b | characteristic point of a fuzzy set carrier |
| **C** | fuzzy constraint |
| c | characteristic point of a fuzzy set carrier |
| d | characteristic point of a fuzzy set carrier |
| dV/dt | acceleration |
| $d\Theta/dt$ | flight path angle rate |
| $d\Psi/dt$ | heading angle rate |
| **G** | goal set of fuzzy situations |
| H | flight altitude |
| *i* | FSTN construction step number |
| I | fuzzy set index (fuzzy set-value number) |
| N(**S**) | number of fuzzy situations in the FSTN |
| N(**T**) | number of fuzzy transitions |
| $n_x$ | longitudinal load factor |
| $n_z$ | normal load factor |
| q | dynamic pressure |
| **S** | fuzzy flight situation |
| t | flight time |
| **T** | fuzzy transition |
| $T_c$ | temperature at critical point |
| U | control space |
| $\underline{U}$ | fuzzy measurement scale in U space |
| u | numeric control variable |
| $\underline{u}$ | linguistic control variable |
| V | vehicle speed |
| X | state space |
| $\underline{X}$ | fuzzy measurement scale in X space |
| $\underline{x}$ | linguistic state variable |
| x | numeric state variable |
| $x_{inf}$, $x_{sup}$ | lower and upper limits of a fuzzy scale carrier |
| y | auxiliary state vector |
| AoA | angle of attack |
| $\Delta$ | transition duration |
| $\Delta\mathbf{V}$ | thrust impulse |
| $\Delta\rho$ | deviation of the atmospheric air density |
| $\Phi$ | operational factor space |
| $\Theta$ | flight path angle |
| $\Psi$ | heading angle |
| $\alpha$ | angle of attack |
| $\delta$ | fuzzy affinity vector, $\delta = (\delta^1, \ldots, \delta^9)$ |
| $\gamma$ | bank angle |
| $\varphi$ | latitude |
| $\lambda$ | longitude |
| $\mu$ | average membership of a target situation $S(t+\Delta)$ |
| $\mu_C(\underline{x}(t))$ | degree of compatibility of a fuzzy state $\underline{x}(t)$ and a fuzzy constraint C |
| $\mu_C(\sigma_k)$ | degree of compatibility of a fuzzy set-value $\sigma_k$ and a fuzzy constraint C |
| $\sigma$ | fuzzy set-value |
| $\chi$ | fuzzy scale irregularity parameter |

## APPENDIX.
## SOME RESULTS
## OF SIMULATION EXPERIMENTS

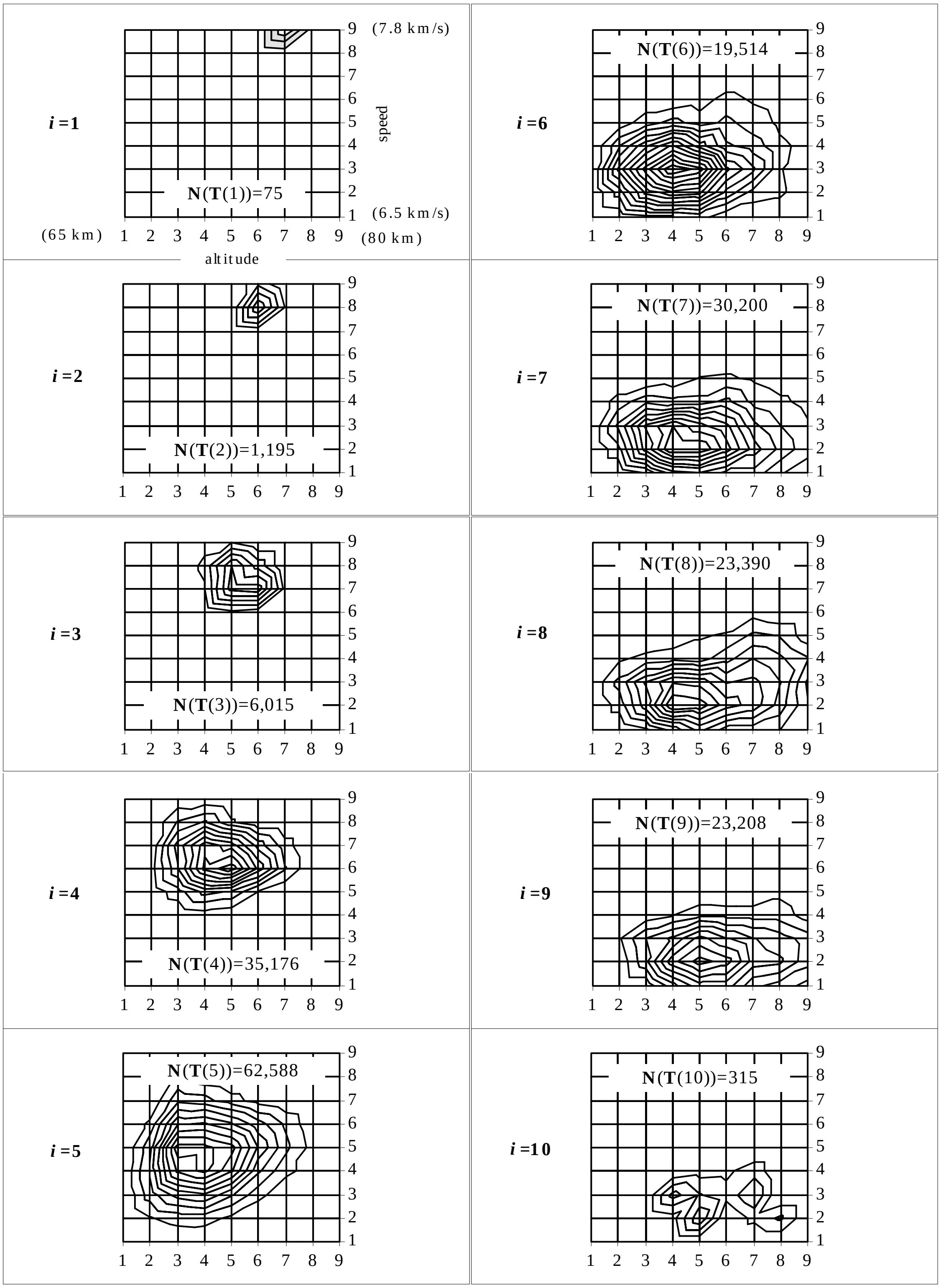


**Fig. 5**. Evolution of subsets of generated fuzzy transitions
in projection on the 'altitude-speed' phase plane during FSTN construction

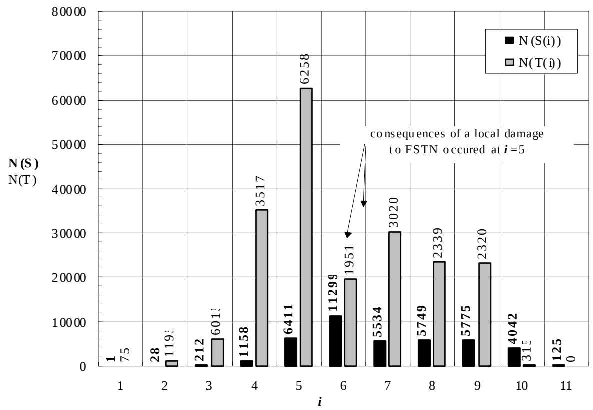


**Fig. 6**. Number of source fuzzy situations N(**S**) and generated fuzzy transitions N(**T**) as a function of the FSTN construction step

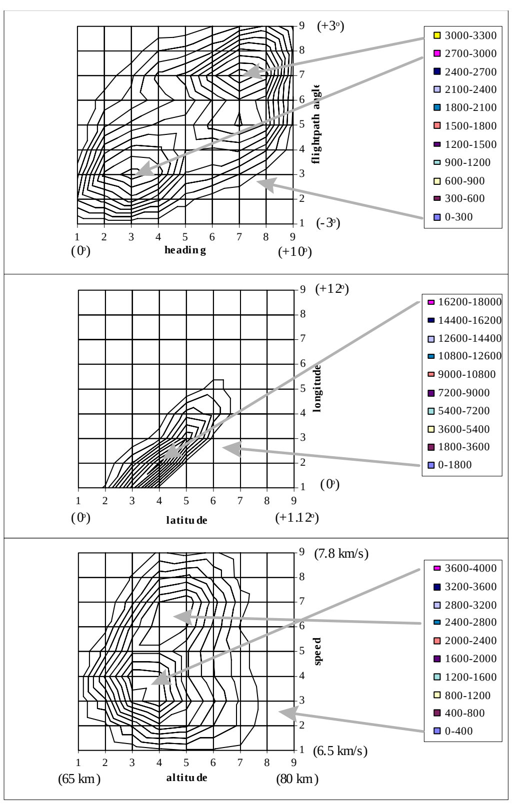


**Fig. 7**. Projection of target situations of 73,287 fuzzy transitions on various phase planes (a subtree from the FSTN for the air density hypothesis ‘Δρ=2’)

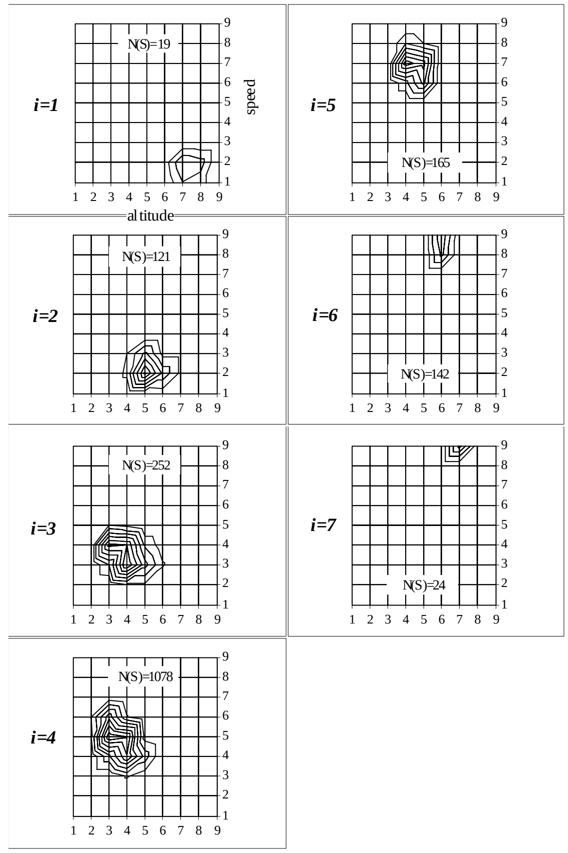


**Fig. 8**. Evolution of source situations of optimum transitions during optimization (‘altitude-speed’ phase plane, air density hypothesis ‘Δρ=4264’)

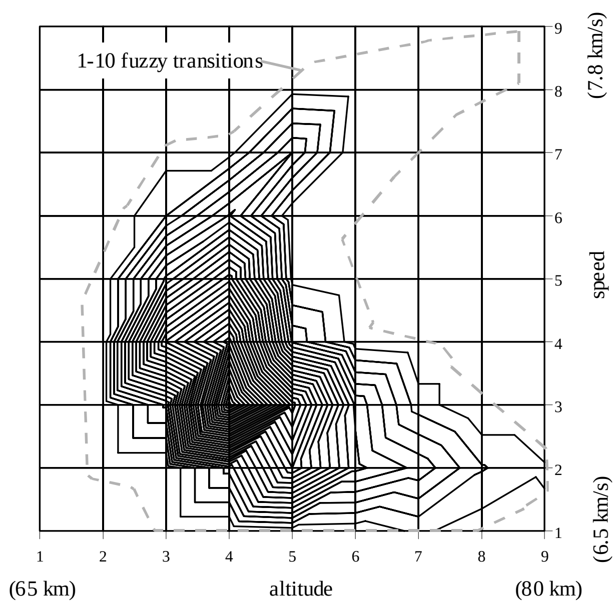


**Fig. 9**. Projection of 1,801 optimized fuzzy transitions on the plane ‘altitude-speed’ of their source situations (hypotesis ‘Δρ=4264’)

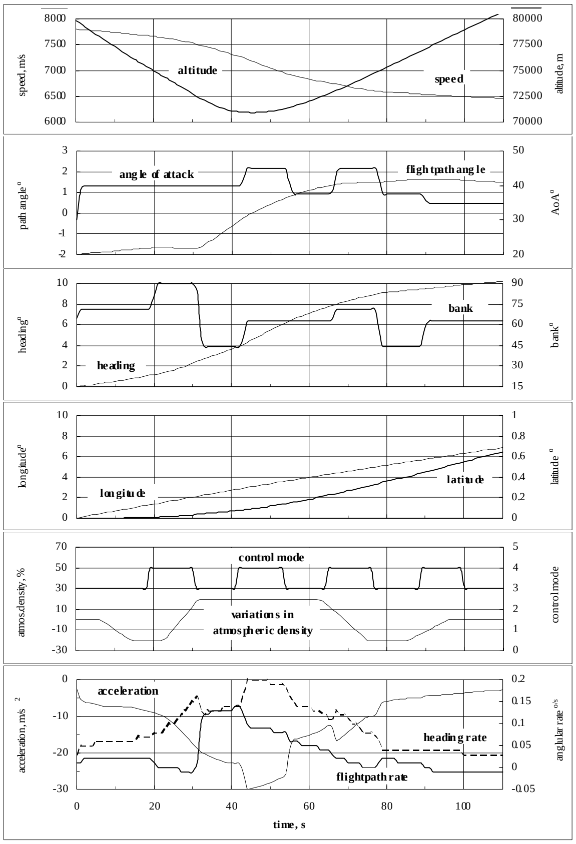


**Fig. 10**. High-altitude hypersonic maneuvering of a transtmospheric vehicle (Flight 207, density hypothesis ‘$\Delta\rho$=4264’, $[\alpha_{INF};\alpha_{SUP}]$=[25º;45º], minimum weight)

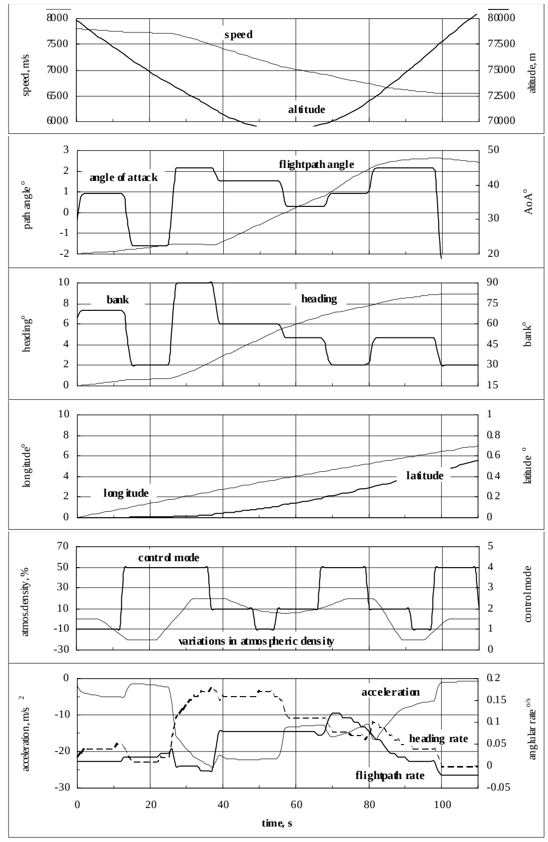


**Fig. 11**. High-altitude hypersonic maneuvering of a transtmospheric vehicle (Flight 120, density hypothesis ‘$\Delta\rho$=4264’, $\delta$=(011 1212 11), maximum weight)